Available online at www.sciencedirect.com
SCIENCE DIRECT®

Composites: Part B 34 (2003) 273–284

composites
Part B: engineering

www.elsevier.com/locate/compositesb

Optimum design of fiber-reinforced composite cylindrical skirts for solid rocket cases subjected to buckling and overstressing constraints

Cho-Chung Liang[a,*], Hung-Wen Chen[b]

[a]Department of Mechanical Engineering, Da-Yeh University, 112. Shan-Jiau Rd., Da-Tsuen, Changhua 515, Taiwan, ROC
[b]Institute of System Engineering, Chung Cheng Institute of Technology, Ta-Shi, Tao-Yuan 335, Taiwan, ROC

Received 6 March 2002; accepted 24 August 2002

Abstract

In order to increase the flight range of aerospace vehicles and the efficiency of solid rocket motors, designers attempt to reduce the weight of solid rocket motors. A skirt is a potential element for weight reduction in rocket motors as it leads to reduction of the total weight of solid rocket motor. Due to its significance for solid rocket motors, the objective of this paper is to investigate the optimal design of a fiber-reinforced composite cylindrical skirt subjected to a buckling strength constraint and an overstressing strength constraint under aerodynamic torque and axial thrust. The present optimal design problem involve in determining the best laminate configuration to minimize the weight of the cylindrical skirt. To find the optimal solution accurately and quickly, the hybrid genetic algorithm (HGA) is employed in this work. Buckling strength and overstressing strength of the fiber-reinforced composite cylindrical skirt are analyzed using classical laminate theory and elastic stability theory of thin shells. The Tsai-Wu failure criterion is employed to assess the first ply failure, and an overstressing load level factor is introduced to describe the failure strength. In addition, a buckling load factor is introduced to describe the buckling strength. Due to the critical issue of buckling strength, the effects of the design parameters on the buckling strength are investigated in this work. Finally, a practical design example of the proposed fiber-reinforced composite cylindrical skirt is investigated using the present analysis procedure. Results reveal that the fiber-reinforced composite cylindrical skirt laminated symmetrically with both cross-ply layers [0/90°] and angle-ply layers [+45/ − 45°] can sustain a great buckling load. Furthermore, the buckling strength of the skirt shell laminated with equal-hybrid between the angle-ply layers and the cross-ply layers is greater than that of the skirt shell laminated with over-weighted hybrid between the angle-ply layers and the cross-ply layers. Results provide a valuable reference for designers of aerospace vehicles.

Keywords: Skirt; Fiber-reinforced; Angle-ply; Cross-ply; Buckling; Overstressing

1. Introduction

A skirt is a key element of a rocket motor as shown in Fig. 1 and is constructed using a circular cylinder at the tangent zones where the domes and cylinders meet. From the standpoints of structural integrity, compatibility, cost and ease of fabrication, a fiber-reinforced composite skirt, instead of the conventional steel one, is desirable. It is wound or laid up using temporary skirt tooling, which is removed following curing. The fiber-reinforced composite skirt is attached using various bonding/winding or riveting method to convey loads through the motor case assembly. In order to satisfy the strength requirement, the fiber-reinforced composite skirt is usually a laminated cylinder shell consisting of cross-ply layers and angle-ply layers. The cross-ply layers with the layers oriented at 0 or 90° only are arranged in the outer of the shell, and the angle-ply layers with an equal number of layers oriented at $\pm\theta°$ angles are arranged in the inner part of the shell. In service, the skirt of solid rocket motor is commonly subjected to aerodynamic torque and axial thrust. Buckling due to combined loads and failure due to overstressing are two major concerns in the safe and reliable design of the skirt. Furthermore, to meet the system needs of aerospace vehicles, designers also attempt to reduce the weight of the solid rocket motor. Reducing its weight results in increased flight range of the vehicles and increased efficiency of the solid rocket motor. The skirt is one of the potential elements for weight reduction in rocket motors as it leads to a reduction in the total weight of the solid rocket motor. Weight reduction of the skirt is achieved not only by using fiber-reinforced

* Corresponding author. Tel.: +886-4-8511220; fax: +886-4-8511224.
E-mail address: ccliang@mail.dyu.edu.tw (C.-C. Liang).

1359-8368/03/$ - see front matter
PII: S1359-8368(02)00104-X

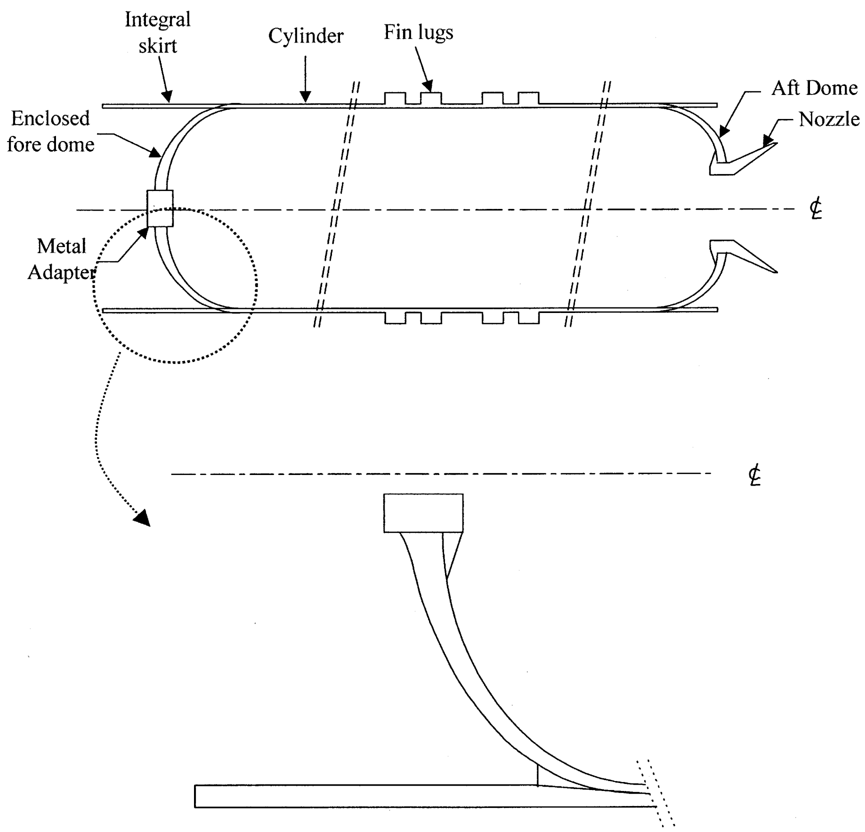


Fig. 1. A schematic diagram of a circular cylindrical skirt on a rocket motor case.

composite materials, but also by improving the design of the lamination configuration. Because fiber-reinforced composites have high elastic strain energy storage capacity and high strength-to-weight ratio compared with steel, it is possible to reduce the weight of a skirt without reducing the load carrying capacity. Moreover, through the design of various lamination parameters, such as ply orientations and the thicknesses of layers, the structural performance of a fiber-reinforced composite skirt can be upgraded. Therefore, in this work, a ply orientation and thicknesses are adopted as design parameters to minimize the weight of a fiber-reinforced composite skirt for solid rocket motor cases subjected to structural stability and strength constraints. This study is available in the preliminary design of solid rocket motor.

The literature on skirt design is very limited. The few reports on the design of solid rocket motors have merely described the purposes of skirt for solid rocket motors and briefly presented manufacturing processes and design concepts of skirts. Francis Rosato and Grove [1] investigated the development, manufacture, and design of skirts for filament winding composite rocket motors. Jame's report [2] on structural design and analysis of filament wound rocket motor cases described loads reacting on the skirt in static firings. Maheshwari and Grover [3] presented the development of skirts for an advanced composite rocket motor case under internal and external loads. They also presented methods for forming and reinforcing manner on skirts. Hoffman's report [4] on rocket motor cases described external loadings on skirts during flight, along with designs, fabrications and compositions of skirts for various rocket motor cases. Peter et al. [5] briefly described skirt fabrication for filament winding composite rocket motors in their book. Francis [6] presented a variety of technical challenges encountered in the development of the ERINT-1 motor case, which include the integral of skirts. He discussed the design and construction requirements of integral skirts and the selection of usable materials. In the design, the skirt can be treated as a cylinder shell structure. In past years, several major works on the subject of the optimum design of composite cylinder shells have been proposed. In 1983, Nshanian and Pappas [7] proposed a method based on a mathematical programming algorithm to identify the optimal ply angle variation across the thickness of laminated cylindrical shells and to determine the nature of the improvement in performance that can be expected from optimal configurations. In 1985, Onoda [8] applied an energy method to obtain the optimal laminate configurations of cylindrical shells under an axial buckling load and investigated the relationship between the change of the buckling load and the change in the laminate parameter values. In 1988, Sun and Hansen [9] adopted lamina fiber orientations as optimizing parameters to maximize the buckling load of a laminated-composite circular-cylindrical shells subjected to combined loads. In 1989, Sun [10] investigated the effects of ply stacking sequencing and the angle on the cylinder buckling pressure using a random search procedure based on Powell's optimization method. In 1991, Hu [11] used a sequential linear programming method together with a simple move-limit strategy to maximize the buckling strength of fiber-composite laminate shells with a given loading condition and material system with respect to fiber orientations and investigated the effects of the shell geometry on the buckling strength. In 1995, Zimmermann [12] used a mathematical programming method based on coordinate pattern search to maximize the buckling load of fiber composite cylindrical shells subjected to axial compression with respect to fiber orientations. In 1996, Xie et al.[13] described a method of analyzing the maximum buckling strength of hybrid-fiber multiplayer-sandwich cylindrical shells under external lateral pressure with respect to fiber orientations and weighting factors. In 1997, Walker et al. [14] determined the best layup to minimize the weight for a hybrid laminated cylindrical shell subjected to an axial buckling load constraint using an analytical technique based on symbolic computation. In 1997, Walker et al. [15] determined the best layup to optimize the multiobjective design of laminated cylindrical shells for a weighted combination of maximum axial and torsional buckling loads. In 2000, Smerdov [16,17] dealt with different formulations of optimization problems on buckling of multi-layered composite cylindrical shells under axial compression and external pressure as well as the effects of variations in the dimensions and materials of the shell on buckling strengths. In 2001, Adali et al. [18] tried to minimize the sensitivity of laminated shells to variations in ply angles subject to a constraint on the buckling load and computed the optimal ply angles to achieve this objective. In the above studies, investigations on cylinder shells mainly used mathematical programming technology to solve problems and complete lamination configuration designs. However, the fiber-reinforced composite cylindrical skirt is a complex cylinder due to its laminate structure and applications. From the analysis viewpoint, it is a complicated design problem. The mathematical programming technologies that have been employed in previous investigations are apt to fall into the local optimum for this type of nonlinear problem. Therefore, in this paper, we attempt to optimize the design of a fiber-reinforced composite cylindrical skirt subjected to the stability constraint and the failure strength constraint using hybrid random search algorithms, specifically, the combined genetic search and Powell's search methods.

In engineering optimization, calculus-based techniques, such as nonlinear programming, are mathematical programming-based methods that search for an extreme guided by the gradient of the objective function at different points in the search space. These techniques may not find the gradients for non-convex constraints or may fall into the local optimum for nonlinear problems with multiple local extremes [19]. The optimization design problem of a fiber-reinforced composite cylindrical skirt, referred to as the nonlinear constrained problem, is complex in nature and difficult to solve using these calculus-based optimization techniques. Thus, two random search optimization techniques based on significantly different principles are hybridized and used here to avoid calculating the gradients. One is genetic algorithm (GA), which is an artificial adaptive random search technique. This heuristic genetic search technique finds the optimal designs based on the mechanism of natural selection and natural genetics [20,21]. Another is Powell's search method, which is hill-climbing algorithm. This zero-order optimization method uses the concept of conjugate search directions to find the optimal solution [22,23]. The GA can determine good regions within the search space quickly. However, like the evolution of life, it is very slow at finding the optimal design within that region. Powell's search method, in contrast, is fast but has a tendency to get trapped on the local extreme and thus fails to find the true optimum. In this paper, a hybrid genetic algorithm (HGA) [24] is used to search for the optimum design of a fiber-reinforced cylindrical skirt. This HGA starts with the initial global search of the GA to find good solution areas. Once the GA finishes, the final solution is used as the starting value for an optimization run using the Powell's search method. This hybrid algorithm is used to overcome the difficulty that the GA has in climbing the last slope to the optimal point and to avoid the weakness that Powell's method has in single point search [24].

The objective of this study is to minimize the weight of a fiber-reinforced composite cylindrical skirt subjected to a buckling strength constraint and an overstressing strength constraint under aerodynamic torque and axial thrust using the HGA. Design variables are the orientation angle of the angle-ply layers, the total thickness of the cross-ply layers in the outer part of the cylindrical skirt shell, and the total thickness of the angle-ply layers in the inner part of the cylindrical skirt shell. Due to the critical issue of the buckling strength, the effects of the design parameters on the buckling strength are also investigated in this work. Finally, a practical design example is presented and investigated using this design procedure. Some good results presented herein provide a valuable reference for designers of aerospace vehicles. In addition, the procedure employed in this paper can be used as a design tool in the preliminary stage of designing complex fiber-reinforced laminated cylindrical structures for various applications.

2. Buckling and failure analysis of a fiber-reinforced composite cylindrical skirt

A fiber-reinforced composite cylindrical skirt sustains torque due to the aerodynamic characteristics of the fin and axial compression due to the thrust of rocket motor in flight. When these loads exceed critical values, buckling and overstressing occur in the skirt. To understand the mechanical behaviors of a fiber-reinforced composite cylindrical skirt and to avoid these failures, the objective of this section is to explicitly describe buckling and failure strength analysis of a fiber-reinforced composite cylindrical skirt using the classical laminate theory [25,26] and the elastic stability theory [27] of thin shells.

In this study, a fiber-reinforced composite cylindrical skirt of length L, radius R (from the center to the mid-surface of the shell), and total thickness h under aerodynamic torque T and axial thrust F is considered and investigated, as shown in Fig. 2. The shell has symmetric and balanced laminates. Let the mid-surface of the cylindrical shell of the skirt be the reference surface, and let the origin of the coordinates be located at one end of the cylinder. The orthogonal coordinates x, y, and z are measured in the longitudinal, circumferential, and radial directions, respectively. The fiber angle is defined as the angle between the fiber direction and the longitudinal axis. The fiber orientations, which include the angle-ply layers ($[\pm\theta]$) in the inner part of shell and the cross-ply layers ($[90/0°]_i$) in the outer part of shell, are symmetric with respect to the mid-surface of the cylindrical shell (see Fig. 2).

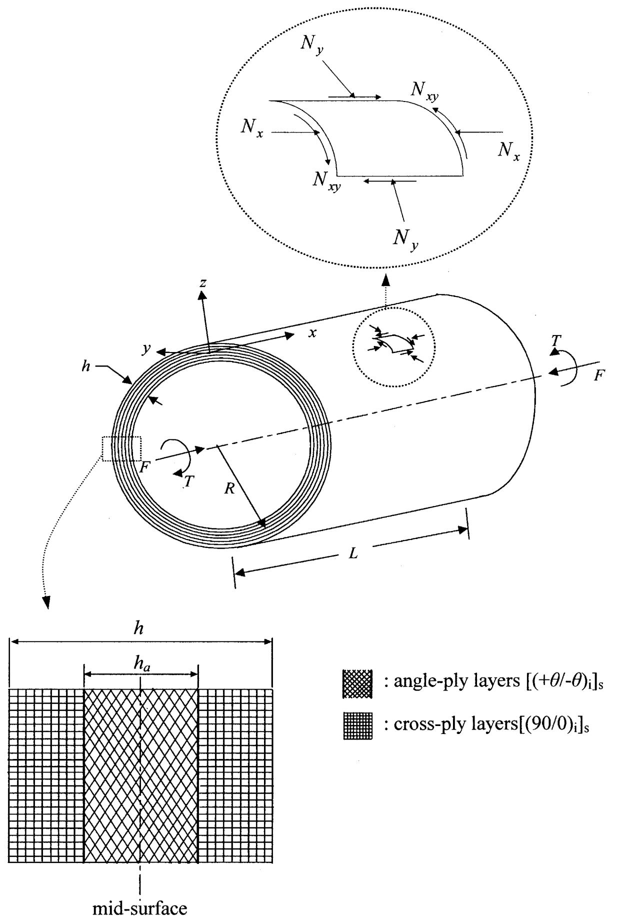


Fig. 2. The shell elements and resultant forces of the circular cylindrical skirt.

When the fiber-reinforced composite cylindrical skirt is subjected to aerodynamic torque T and axial thrust F, the force and moment resultants (see Fig. 2) are

$$N_x = \frac{F}{2\pi R}, \qquad N_{xy} = \frac{T}{2\pi R^2}, \qquad (1)$$
$$N_y = M_x = M_y = M_{xy} = 0.$$

The above are the applied loads on the skirt. If the applied aerodynamic torque and axial thrust equal or exceed critical values, buckling or overstressing will occur, which for most practical purposes is synonymous with collapse and failure.

2.1. Lamination theory for a fiber-reinforced composite cylindrical skirt

This section will describe the mechanics behaviors of a fiber-reinforced composite cylindrical skirt that has particular lamination configuration, using the classical lamination theory. Since the shell of the fiber-reinforced composite cylindrical skirt is a symmetric laminate with respect to the mid-surface and is balanced, the bending-extensional coupling stiffness matrix $[B]$ can be neglected, and the constitutive equation for the fiber-reinforced composite cylindrical skirt can be rewritten as

$$\{N\}_{xy} = [A]\{\epsilon\}_{xy}, \qquad \{M\}_{xy} = [D]\{\kappa\}_{xy}, \qquad (2)$$

where $\{N\}_{xy}$ is the force resultant tensor, $\{M\}_{xy}$ the moment resultant tensor, $\{\epsilon\}_{xy}$ the strain tensor, $\{\kappa\}_{xy}$ the curvature tensor, $[A]$ the extensional stiffness matrix and $[D]$ the bending stiffness matrix.

For a fiber-reinforced composite skirt shell consisting of cross-ply layers and angle-ply layers, the extensional stiffness matrix $[A]$ is derived as

$$[A] = h_\theta[\bar{Q}]^{(\theta)} + (h - h_\theta)[\bar{Q}]^{(c)}, \qquad (3)$$

and the bending stiffness matrix $[D]$ is derived as

$$[D] = \frac{h_\theta^3}{12}[\bar{Q}]^{(\theta)} + \frac{(h^3 - h_\theta^3)}{12}[\bar{Q}]^{(c)}. \qquad (4)$$

In Eqs. (3) and (4), h_θ denotes the thickness of the angle-ply layers ($[\pm\theta°]_i$), and h denotes the total thickness of the laminate, respectively (Fig. 2). Matrix $[\bar{Q}]^{(\theta)}$ denotes the reduced stiffness of the angle-ply layers of the laminate, and matrix $[\bar{Q}]^{(c)}$ denotes the reduced stiffness of the cross-ply layers of the laminate. The components of these reduced stiffness matrices are described in Appendix A.

In Eqs. (3) and (4), the extension stiffness matrix $[A]$ and bending stiffness matrix $[D]$, which include the design parameters, are the major factors impacting on the stability and strength design of the fiber-reinforced composite cylindrical skirt. These matrices will be used in buckling and failure analysis of the skirt.

2.2. Failure due to overstressing

To assess the capability of the fiber-reinforced composite cylindrical skirt to withstand failure due to overstressing, the Tsai-Wu failure criterion is employed. In addition, we introduce a strength failure load factor ζ to identify the characteristics of the first-ply failure of the cylindrical skirt. Based on the Tsai-Wu failure criterion, the strength level factor ζ is defined as

$$\zeta = \max_k [F_{11}(\sigma_{\mathrm{L}}^{(k)})^2 + F_{22}(\sigma_{\mathrm{T}}^{(k)})^2 + F_{66}(\sigma_{\mathrm{LT}}^{(k)})^2 + 2F_{12}\sigma_{\mathrm{T}}^{(k)}\sigma_{\mathrm{L}}^{(k)} + F_1\sigma_{\mathrm{L}}^{(k)} + F_2\sigma_{\mathrm{T}}^{(k)}]. \quad (5)$$

In Eq. (5), each stress component of the kth layer $\sigma_{\mathrm{L}}^{(k)}$, $\sigma_{\mathrm{T}}^{(k)}$, $\sigma_{\mathrm{LT}}^{(k)}$ in the material direction can be calculated by

$$\{\sigma\}_{\mathrm{LT}}^{(k)} = [T][\bar{Q}]^{(k)}[A]^{-1}\{N\}_{xy}, \quad (6)$$

where $\{N\}_{xy}$ is the resultant tensor $[N_x, N_y, N_{xy}]^{\mathrm{T}}$, shown as Eq. (1); $[A]^{-1}$ the inverse of the extension stiffness matrix $[A]$, shown as Eq. (3); $[\bar{Q}]^{(k)}$ is the reduced stiffness of the kth layer with the laminate; and the strength parameters F_{11}, F_{22} $F66$, F_1 and F_2 are given by

$$F_{11} = \frac{1}{\sigma_{\mathrm{LU}}\sigma'_{\mathrm{LU}}}, \qquad F_{22} = \frac{1}{\sigma_{\mathrm{TU}}\sigma'_{\mathrm{TU}}}, \qquad F_{66} = \frac{1}{\sigma_{\mathrm{TLU}}^2},$$

$$F_1 = \frac{1}{\sigma_{\mathrm{LU}}} - \frac{1}{\sigma'_{\mathrm{LU}}}, \qquad F_2 = \frac{1}{\sigma_{\mathrm{TU}}} - \frac{1}{\sigma'_{\mathrm{TU}}},$$

$$F_{12} = -\frac{1}{2}\frac{1}{\sqrt{\sigma_{\mathrm{LU}}\sigma'_{\mathrm{LU}}\sigma_{\mathrm{TU}}\sigma'_{\mathrm{TU}}}},$$

where σ_{LU}, σ'_{LU}, σ_{TU}, and $\sigma'_{\mathrm{TU}}$are the tensile and compressive strengths of the composite material in the longitudinal and transverse directions, and σ_{TLU} is the in-plane shear strength.

To ensure that first-ply failure does not occur in the cylindrical skirt, the following condition must be satisfied

$$\zeta \le 1. \quad (7)$$

Eq. (7) is used as a restriction on the optimization design problem in this work.

2.3. Buckling loads

The fiber-reinforced composite cylindrical skirt is subjected to two major external loads, which are torque due to aerodynamics and axial compression due to thrust, during flight. The skirt will buckle and, thus, the rocket will fail if these loads exceed critical values. To ensure that the skirt is stable, it is necessary for the critical buckling load to be higher than the actual load in the fiber-reinforced composite cylindrical skirt. In the present study, the buckling load is analyzed using the linear elastic theory of thin shells.

Here, we introduce a scalar multiple λ of the design load, the so-called buckling load factor. It is employed to identify pre-buckling of the fiber-reinforced composite cylindrical skirt and is defined as

$$\lambda = \frac{N_1}{N_{\mathrm{a}}}, \quad (8)$$

where N_{a} is the actual load and N_1 is the structural pre-buckling strength.

In this paper, the buckling load factor λ is used to describe the buckling characteristics of the cylindrical skirt as follows:

1. when $0 \le \lambda < 1$, this indicates that buckling of the cylindrical skirt will occur at the actual load N_{a};
2. when $\lambda = 1$, this indicates that buckling is incipient under the actual load N_{a}; and
3. when $\lambda > 1$, this indicates that buckling will occur at a load level higher than the actual load N_{a}.

2.3.1. Buckling due to axial thrust

Buckling of the shell of a cylindrical skirt constructed of a symmetrical and balanced laminate under axial thrust F (see Fig. 2) is governed by the following system of equations [10]

$$A_{11}u_{,xx} + A_{12}\left(v_{,xy} + \frac{w_{,x}}{R}\right) + A_{66}(v_{,xy} + u_{,yy}) = 0,$$

$$A_{66}(v_{,xx} + u_{,xy}) + A_{12}u_{,xy} + A_{22}\left(v_{,yy} + \frac{w_{,y}}{R}\right) = 0,$$

$$D_{11}w_{,xxxx} + 2(D_{12} + 2D_{66})w_{,xxyy} + A_{12}\frac{u_{,x}}{R} + A_{22}\left(\frac{v_{,y}}{R} + \frac{w}{R^2}\right) + D_{22}w_{,yyyy} = Fw_{,xx}, \quad (9)$$

where u, v, and w are the displacements in the x, y, and z directions, respectively; A_{ij} $(i,j = 1,2,6)$ are the components of the laminate extension stiffness matrix (see Eq. (3)); D_{ij} $(i,j = 1,2,6)$ are the components of the laminate bending stiffness matrix (see Eq. (4)); a subscript comma indicates partial differentiation. It is noted that the bending-extension coupling matrix $[B]$ does not appear in Eq. (9) because the laminate for the skirt shell is symmetrical to the mid-surface.

Consider the installation of a static test of the cylindrical skirt in the preliminary design; a simply supported shell boundary condition (that is: $w = v = 0$ at $x = 0$, L, are satisfied) is restricted. The solution to Eq. (9) is obtained by taking displacements in the form

$$u = u_{nm}\cos(\bar{m}x)\cos(\bar{n}y), \qquad v = v_{nm}\sin(\bar{m}x)\sin(\bar{n}y), \qquad w = w_{nm}\sin(\bar{m}x)\cos(\bar{n}y), \quad (10)$$

where $\bar{m} = m\pi/L$, $\bar{n} = n/R$; m is the number of half-waves in the x-direction; n is the number of waves in the y-direction; u_{nm}, v_{nm} and w_{nm} are the buckling displacement amplitude coefficients. Substituting Eq. (10) into Eq. (9) and combining Eq. (8) and Eq. (1), the buckling load factor for the axial thrust F, λ_{a}, can be derived as the solution of an eigenproblem in the form [28]

$$\lambda_a(m,n;\theta) = \frac{2R}{\pi F}\left(\frac{L}{m}\right)^2 \frac{\begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix}}{\begin{vmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{vmatrix}}, \tag{11}$$

where

$$C_{11} = A_{11}\bar{m}^2 + A_{66}\bar{n}^2,$$

$$C_{22} = A_{22}\bar{n}^2 + A_{66}\bar{m}^2,$$

$$C_{33} = D_{11}\bar{m}^4 + 2(D_{12} + 2D_{66})\bar{m}^2\bar{n}^2 + D_{22}\bar{n}^4 + \frac{A_{22}}{R^2},$$

$$C_{12} = C_{21} = (A_{12} + A_{66})\bar{m}\bar{n},$$

$$C_{23} = C_{32} = \frac{A_{22}}{R}\bar{n},$$

$$C_{13} = C_{31} = \frac{A_{12}}{R}\bar{m}.$$

To determine the critical buckling load factor due to the axial thrust, λ_a^{cr},, we vary the integers m and n, and calculate the minimum value of λ_a. The critical buckling load factor can be stated as

$$\lambda_a^{cr} = \min_{m,n} \lambda_a(m,n;\theta). \tag{12}$$

2.3.2. Buckling due to aerodynamic torque

Due to the good agreement achieved in Ref. [28] between linear theory and experimental results for a buckling of the cylindrical shell under torsion, in this paper, using these experimental results, the critical torsion buckling load for the symmetrical skirt shell is given as [28]

$$T_{cr} = 21.75(D_{22})^{5/8}\left(\frac{A_{11}A_{22} - A_{12}^2}{A_{22}}\right)^{3/8}\frac{R^{5/4}}{L^{1/2}}. \tag{13}$$

Another restriction on this equation is

$$\left(\frac{D_{22}}{D_{11}}\right)^{5/6}\left(\frac{A_{11}A_{22} - A_{12}^2}{12A_{22}D_{11}}\right)^{1/2}\frac{L^2}{R} \geq 500. \tag{14}$$

Substituting Eq. (13) into Eq. (8) and combining this with Eq. (1), the critical torsion buckling load factor λ_T^{cr} for the symmetrical skirt shell under aerodynamic torque Tcan be stated as

$$\lambda_T^{cr} = \frac{T_{cr}}{T} = \frac{21.75}{T}(D_{22})^{5/8}\left(\frac{A_{11}A_{22} - A_{12}^2}{A_{22}}\right)^{3/8}\frac{R^{5/4}}{L^{1/2}}. \tag{15}$$

2.3.3. Buckling due to combined axial thrust and aerodynamic torque

In the preliminary design of the fiber-reinforced composite cylindrical skirt, a combined buckling load factor λ is employed to describe the buckling of the shell subjected to combined axial thrust and aerodynamic torque. λ can be determined by [28]

$$\frac{1}{\lambda} = \frac{1}{\lambda_a^{cr}} + \frac{1}{\lambda_T^{cr}}. \tag{16}$$

The critical buckling load factors λ_a^{cr} and λ_T^{cr} in Eq. (16) are calculated using Eqs. (13) and (15), respectively. To ensure that the fiber-reinforced composite cylindrical skirt subjected to the combined loads does not buckle, the combined buckling load factor λ cannot be less than one. That is,

$$\lambda \geq 1. \tag{17}$$

Eq. (17) serves as another restriction on the optimization design problem in this work.

3. Optimum design of the fiber-reinforced composite cylindrical skirt

This section formulates the optimization design problem of the fiber-reinforced composite cylindrical skirt for solid rocket motors that sustain aerodynamic torque and compressive force. The design problem involves determining the laminate configuration to minimize the weight of the cylindrical skirt subjected to composite strength and structural stability constraints. In this study, the fiber angle and the thicknesses of the layers are used to describe the laminate configuration of the fiber-reinforced composite cylindrical skirt. First-ply failure, treated as a strength constraint, is considered using the Tsai-Wu criterion. Buckling failure due to combined torque and axial compression, treated as a stability constraint, is considered using buckling load factor greater than one ($\lambda > 1$). The HGA is adopted to solve the optimization design problem.

3.1. Optimal problem formulation

The purpose of the formulation is to create a mathematical model of the optimal design problem of the fiber-reinforced composite cylindrical skirt for solid rocket motors that can be solved using the HGA. The mathematical model of the optimization design that includes the design variables, objective function and design constraints are described below.

3.1.1. Design variables

Some parameters usually remain fixed in relation to design variables. Here, the pre-assigned design parameters are the length (L) and radius (R) of the cylindrical skirt; the design loads, which include the torque (T) and compression (F); and material properties.

The thickness (h_c) of the cross-ply layers, the thickness (h_θ) of the angle-ply layers and the orientation angle (θ) of the angle-ply layers are taken as design variables for the optimization problem. Here, they are employed to describe the laminate configuration of the fiber-reinforced composite cylindrical skirt.

3.1.2. Objective function

The objective of the present design problem is to minimize the weight of the fiber-reinforced composite cylindrical skirt for solid rocket motors subjected to overstressing and buckling strength constraints. The objective function is stated as follows

$$\text{Minimize } W = 2\pi RL[\rho_c h + (\rho_\theta - \rho_c)h_\theta], \tag{18}$$

where ρ_c the material density of the cross-ply layers in the outer part of the laminate; ρ_θ the material density of the angle-ply layers in the inner part of the laminate; h the total thickness of the laminate; h_θ the thickness of the angle-ply layers in the outer part of the laminate; L the length of the cylindrical skirt; R the radius of the mid-surface of the cylindrical skirt.

3.1.3. Design constraints

The minimum weight design problem of the fiber-reinforced composite cylindrical skirt treated here has two constraints: buckling strength due to combined axial compression and torsion loads, and failure strength due to overstressing. According to Eqs. (7) and (17), the constraint equations can be rewritten as

$$g_1 : \frac{1}{\lambda} - 1 \le 0, \qquad g_2 : 1 - \frac{1}{\zeta} \le 0. \tag{19}$$

3.1.4. Fitness function

Generally, a GA is used to solve unconstrained optimization problems and to maximize a fitness function. A fitness derived from the objective function serves as an evaluation function in successive genetic operations. In this paper, penalty techniques are used to transform the present constrained optimization problem into an unconstrained problem. In addition, the inverse of the objective function is taken as the fitness. Finally, the fitness function based on the exterior penalty function approach is expressed as

$$\text{Fitness} = \left(\frac{Q}{f(X)}\right) = \left(\frac{Q}{W(X) + r_j \sum_{i=1}^{2} [|g_i(X)| + g_i(X)]^2}\right), \tag{20}$$

where $X = \{h_\theta, h_c, \theta\}$ is the design variable vector; $f(X)$ the modified objective function; $W(X)$ the total weight of the fiber-reinforced composite cylindrical skirt; $r_j \sum_{i=1}^{2} \times [|g_i(X)| + g_i(X)]^2$ a penalty function; Q a normalizing constant; r_j a penalty parameter; j a generation or iteration cycle in the optimization procedure.

3.2. Optimization design procedure based on HGA

To improve the performance of the conventional GA and find the optimal solution accurately, the HGA is presented in this work. The implementation of this algorithm involves a GA, elitism strategy and Powell's search method as shown schematically in Fig. 3.

The design process starts with random selection of a specified number of designs that comprise the initial population for the elitism GA of the HGA. Material properties, the radius and length of the cylindrical skirt, the boundary conditions of the skirt shell and design loadings are input to the analysis processor routines that include buckling analysis and strength failure analysis. Buckling analysis provides the critical buckling load factor, and strength failure analysis provides the strength failure

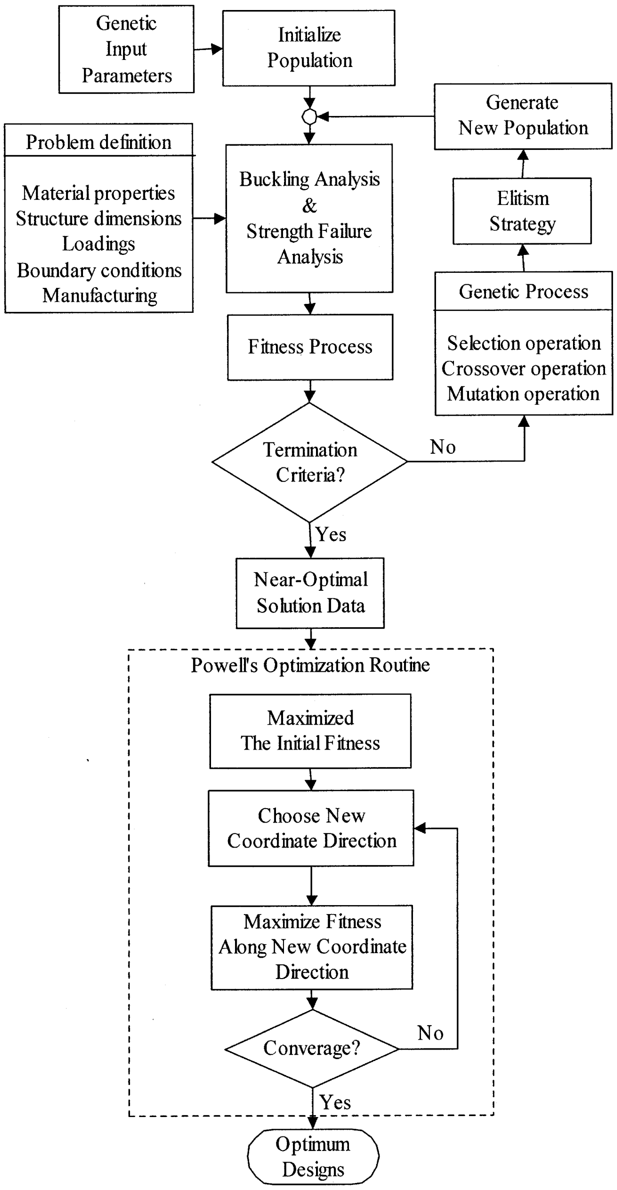


Fig. 3. Flow chart for the optimization procedure using the HGA.

load factor. Then, the weight of the fiber-reinforced composite cylindrical skirt is calculated. This procedure is repeated for each design configuration in the population. The fitness processor evaluates the fitness of each design by employing Eq. (20) and assigns a rank based on an objective function. The evaluated population is then processed. The current population of the design configuration is then processed by the genetic operators and elitist strategy, and a new population of design configurations is created for next generations. This new population combines the most desirable characteristics of previous generations. Designs from previous generations can be replaced by new ones except for the best designs that are always included in the next generation. This GA, which includes the elitist strategy, guarantees that the best designs will occur in the final population. The elitism GA process is repeated until the rate of increase in the best fitness found falls below some arbitrary small. Once this elitism GA finds good design areas in the search pace, the near-optimal design is taken as the initial value for Powell's search algorithm, and the optimal design is searched continually by Powell's optimization run.

Powell's optimization method operates by maximizing the fitness in terms of each variable independently, and forms conjugate maximization paths from each maximized fitness. This search method basically uses a two-part iterative procedure to find the maximum fitness. It starts with an initial estimate of the maximum fitness, and in the first iteration independently maximizes the fitness in each of the coordinate directions to find the new maximum fitness. After this initial maximization, the first coordinate is replaced with a new coordinate direction determined from the initial estimate of the maximum fitness to the new maximum. The next iteration independently maximizes the fitness with respect to the new projected coordinate direction and the remaining coordinates. This procedure is repeated until the fitness undergoes no improvement after all the directions have been tested and the directions tested have not changed. Powell's search procedure is shown in detail in Fig. 4 [22,23].

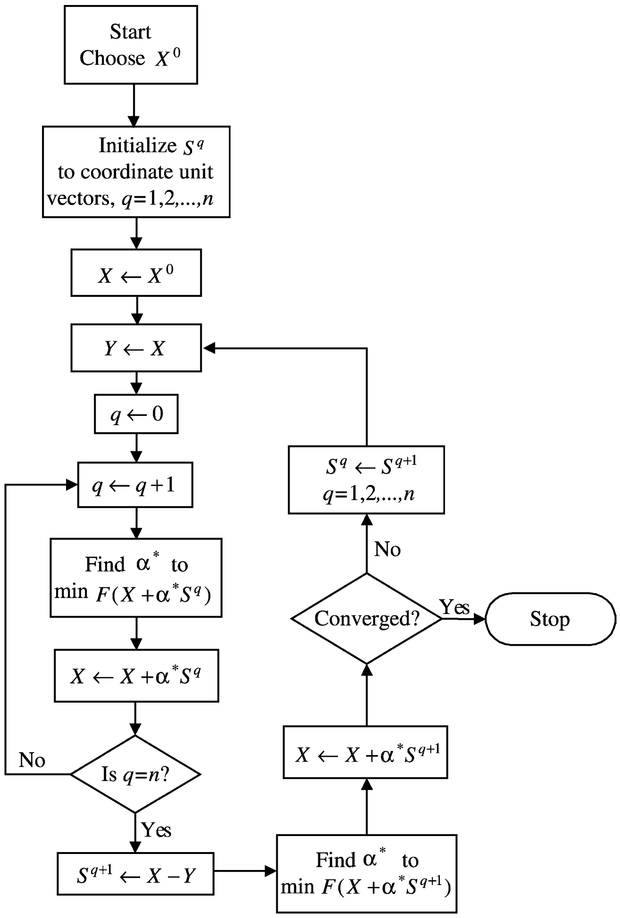


Fig. 4. Flow chart of Powell's search procedure.

4. A practical optimum design example of the fiber-reinforced composite cylindrical skirt for solid rocket cases

In this section, the optimization design problem of a fiber-reinforced composite cylindrical skirt is presented. The design problem aims to obtain the optimum orientation angle of the angle-ply layers and the thickness allocation between the angle-ply layers and cross-ply layers of the skirt shell laminate, which minimize the weight and satisfy buckling and overstressing failure constraints. In addition, the effects of the design parameters on the buckling strength in the design of the fiber-reinforced composite cylindrical skirt subjected to both axial thrust and torque loadings are investigated. A practical example of a fiber-reinforced composite cylindrical skirt is investigated using the present method.

4.1. Model description

The solid rocket motor is a major power source for aerospace vehicles. It burns fuel inside its case, thus generating high pressure and thrusting the vehicle. The fiber-reinforced composite cylindrical skirt sustains both torsional load due to aerodynamics and axial compressive load due to the thrusts of the rocket motor during launch and flight. These loads vary in various situations. In this paper, we consider these practical peak loads acting on fiber-reinforced composite cylindrical skirt. These loads are the axial compressive thrust, $F = 70{,}000$ N and the aerodynamic torque, $T = 70{,}000$ N m. The fiber-reinforced composite cylindrical skirt has a middle radius $R = 0.3$ m and a length $L = 0.5$ m. The material properties used in this study are those of the unidirectional carbon/epoxy composite given by

$$E_L = 137 \text{ GPa}, \quad E_T = 8.17 \text{ GPa}, \quad G_{LT} = 4.75 \text{ GPa},$$

$$\nu_{LT} = 0.316,$$

Table 1
Optimal designs for the present practical design problem with three design variables (θ, h_θ, h_c)

θ (°)	h_θ (mm)	h_c (mm)	h (mm)	Weight (kg)	Fitness	Buckling load factor			Strength level factor (ζ)
						λ_a^{cr}	λ_T^{cr}	λ	
42.5	1.807	1.877	3.684	5.57	17998	1.1778	6.6951	1.0016	0.00030

$$\sigma_{LU} = 1470 \text{ MPa} \qquad \sigma_{TU} = 39.2 \text{ MPa},$$

$$\sigma_{TLU} = 78.4 \text{ MPa}, \qquad \sigma'_{LU} = 980 \text{ MPa},$$

$$\sigma'_{TU} = 78.4 \text{ MPa}.$$

The material constants for a lamina, E_L, E_T, G_{LT}, and ν_{LT}, are the elastic modulus in the longitudinal and transverse directions, the shear modulus and the major Poisson ratio, respectively, based on the material principal coordinate axes. The failure strengths σ_{LU}, σ'_{LU}, σ_{TU}, σ'_{TU} and σ_{TLU} are those defined earlier.

For the present optimization design problem, the objective function is stated in Eq. (18), and the constraint is described in Eq. (19). The thickness of the cross-ply layers (h_c), the thickness of the angle-ply layers (h_θ) and the orientation angle (θ) are taken as design parameters, and their side constraints are described, respectively, as

$$0 \le h_c \le R/10, \qquad 0 \le h_\theta \le R/10,$$
$$(h_c + h_\theta) \le R/10, \quad 0 < \theta < 90. \tag{21}$$

4.2. Optimal design results and discussion

First, optimal results for the present practical design problem of a fiber-reinforced composite cylindrical skirt are shown in Table 1. This table indicates that the optimal design point is $(\theta, h_\theta, h_c) = (42.5°,\ 1.807 \text{ mm},\ 1.877 \text{ mm})$, and that the minimum weight is 5.57 kg. In addition, Table 1 shows that the critical compressive buckling load factor is $\lambda_a^{cr} = 1.1778$, that the critical torsional buckling load factor is $\lambda_T^{cr} = 6.6951$, that the combined buckling load factor is $\lambda = 1.0016$ and that the strength level factor is $\zeta = 0.0003$. These results reveal that the critical axial compressive load is 82,446 N, and that the critical torsional load is 46,8657 N m for the fiber-reinforced composite cylindrical skirt designed using the present method. Because $\lambda \approx 1$ and $\zeta \ll 1$, the buckling strength is the critical design criterion, and the overstressing strength is the relaxative restriction for the present practical optimization design problem.

The optimal design results for the present practical design problem with various pre-assigned orientation angles of the angle-ply layers are shown in Table 2. This table indicates the optimal thickness allocation between the cross-ply layers and the angle-ply layers in the shell laminate of the fiber-reinforced composite cylindrical skirt. The dependence of the optimal thickness designs on the orientation angles of the angle-ply layers is shown in Fig. 5. The curves in Fig. 5 represent the total shell thickness, the thickness of the cross-ply layers and the thickness of angle-ply layers. They are interpolated linearly by the data listed in Table 2. The results show that

1. For the layer-hybridized laminate of the skirt shell, the optimal total shell thickness first decreases and later increases as the orientation angle of the angle-ply layers increases; meanwhile, the optimal shell thickness is $h = 3.684$ mm when the orientation angle of the angle-ply layers is $\theta = 42.5°$. Because the same material is considered in the both cross-ply layers and angle-ply layers, the dependence of the minimum weight on the orientation angle of the angle-ply layers is the same as that for the total shell thickness.

Table 2
Optimal designs for the present practical design problem with various pre-assigned orientation angles of the angle-ply layers

θ (°)	h_θ (mm)	h_c(mm)	h(mm)	Weight (kg)	Fitness	Buckling load factor			Strength level factor (ζ)
						λ_a^{cr}	λ_T^{cr}	λ	
7.5	2.192	2.354	4.546	6.86	14585	1.0837	13.1875	1.0014	0.11570
15	2.034	2.200	4.234	6.38	15664	1.1005	10.9712	1.0002	0.09750
22.5	2.035	1.943	3.978	5.98	16712	1.1298	9.0073	1.0039	0.0694
30	1.997	1.839	3.836	5.78	17289	1.1705	7.8448	1.0185	0.03860
37.5	1.854	1.854	3.708	5.60	17880	1.1692	6.9354	1.0003	0.01390
45	1.438	2.327	3.765	5.68	17611	1.1598	7.3076	1.0010	0.00085
52.5	1.147	2.735	3.882	5.85	17082	1.1455	7.9423	1.0011	0.00030
60	1.593	2.280	3.873	5.84	17122	1.1583	7.3154	1.0000	0.00956
67.5	1.928	2.092	4.020	6.06	16498	1.1549	7.5728	1.0021	0.03380
75	1.985	2.316	4.301	6.49	15418	1.1279	8.9527	1.0017	0.06360
82.5	2.130	2.445	4.575	6.90	14496	1.1075	10.3000	1.0000	0.08780

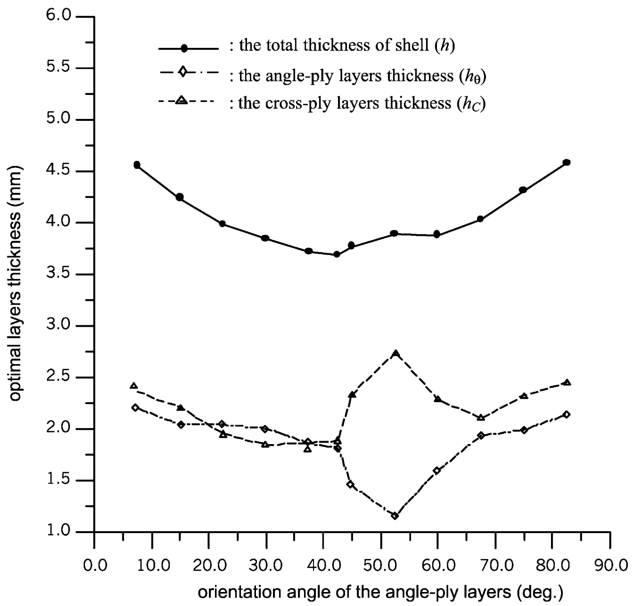


Fig. 5. Optimal thickness designs with various pre-assigned orientation angles of the angle-ply layers.

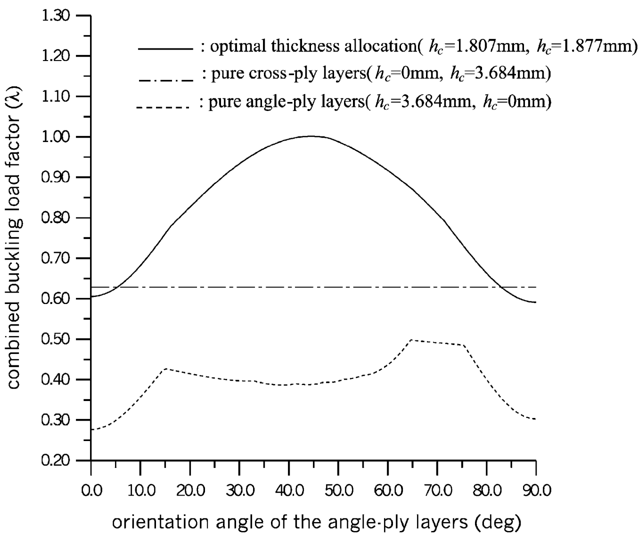


Fig. 6. Combined buckling load factor vs orientation angles of the angle-ply layers for three different thickness allocations between cross-ply layers and angle-ply layers.

2. As for the optimal thickness allocation, the optimal thickness of the angle-ply layers is nearly the same as the optimal thickness of the cross-ply layers when the orientation angle of the angle-ply layers is $\theta \leq 42.5°$ and $\theta \geq 67.5°$. However, the thickness difference between the angle-ply layers and the cross-ply layers is apparent at $42.5° \leq \theta \leq 67.5°$.

4.3. Effects of the design parameters on buckling strength

As indicated by the preceding results, the buckling strength is the critical criterion. In addition, design parameters greatly influence the buckling strength of the fiber-reinforced composite cylindrical skirt. Therefore, the effects of the design parameters on the buckling strength are investigated in this section. These design parameters are the thickness allocation between the angle-ply layers and the cross-ply layers, and the orientation angle of the angle-ply layers.

4.3.1. Effect of the orientation angle of the angle-ply layers on buckling strength

The dependence of the combined buckling load factor on the orientation angle of the angle-ply layers is shown in Fig. 6 for the shell laminate with the optimal total thickness (h), which was 3.684 mm. Fig. 6 shows three thickness allocations for the angle-ply layers and the cross-ply layers in the laminate, including the laminate with the optimal hybrid-layers at $\theta = 42.5°$, the laminate with pure cross-ply layers and that with pure angle-ply layers. In this figure, some results are summarized as

1. For the laminate with pure angle-ply layers ($h_\theta = 3.684$ mm), the curve is piecewise continuous with three peaks of the combined buckling load factor at 15, 65 and 75° for the orientation angle of the angle-ply layers. Because variation of the buckling mode is apparent for the laminate with pure angle-layers, the tendency of the combined buckling load factor is piecewise continuous.
2. For the laminate with pure cross-ply layers ($h_c = 3.684$ mm), the combined buckling load factor stays constant because there are no pure angle-layers in the laminate, and because the laminate stiffness matrixes $[A]$ and $[D]$ remain constant.
3. For the laminate with optimal hybrid-layers, the curve is continuous with a peak value at $\theta = 42.5°$. The results also show that the buckling strength for the laminate with hybrid-layers is greater than those for the laminates with single layers.

4.3.2. Effect of the thickness allocation between the angle-ply layers and cross-ply layers on buckling strength

The dependence of the buckling load factors on the thickness allocation between the angle-ply layers and the cross-ply layers is shown in Fig. 7 for the six pre-assigned orientation angles (15, 30, 45,42.5, 60, and 75°) of the angle-ply layers under the given optimal total shell thickness. For the respective optimal designs with the six pre-assigned orientation angles, the buckling load factors are shown in Table 2. In Fig. 7, the curves present the same phenomena mentioned above: the combined buckling load factor first increases and later decreases as the thickness of the angle-ply layers increases. When the thickness of the angle-ply layers is far smaller than the thickness of the cross-ply layers or the thickness of the angle-ply layers is far larger than the thickness of the cross-ply layers, the combined buckling

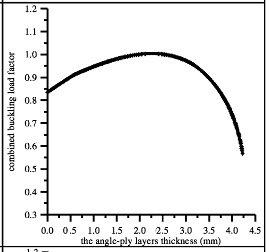
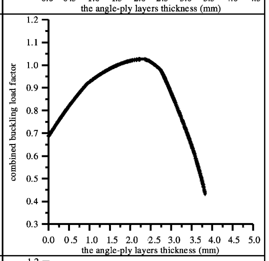
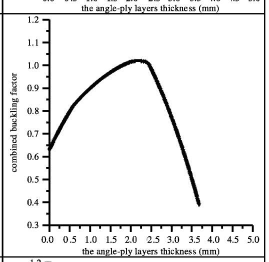
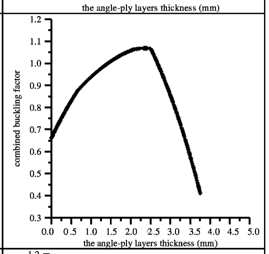
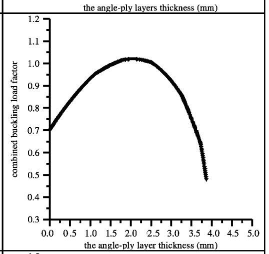
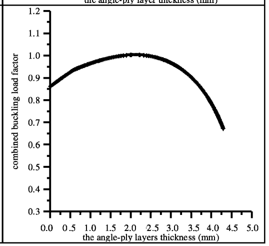
	Optimal thickness (h_θ, h_c, h)	Combined buckling load factor, λ
θ=15°	(2.034,2.200,4.234)	
θ=30°	(1.997,1.854,3.851)	
θ=42.5°	(1.807,1.877,3.684)	
θ=45°	(1.438,2.327,3.765)	
θ=60°	(1.593,2.280,3.873)	
θ=75°	(1.985,2.316,4.301)	

Fig. 7. Combined buckling load factor vs the angle-ply layers thickness given the optimal total thickness of the shell for various pre-assigned orientation angles of the angle-ply layers.

load factor is small. The results also show that the buckling strength for the laminate with equal hybrid between the angle-ply layers and the cross-ply layers is greater than the laminates with over-weighted hybrid between the angle-ply layers and the cross-ply layers.

5. Conclusion

Due to the importance of the skirt in solid rocket motors, this paper has investigated the optimal design of a fiber-reinforced composite cylindrical skirt subjected to a buckling strength constraint and an overstressing strength constraint under aerodynamic torque and axial thrust. The present optimal design problem involves determining the best laminate configuration to minimize the weight of the cylindrical skirt. The orientation angle of the angle-ply layers, the thickness of the cross-ply layers, and the thickness of the angle-ply layers has been considered as design parameters. To find the optimal solution accurately and quickly, the HGA has been employed in this work. It had been proved that the HGA is very efficient method for use in the optimization design procedure [24]. In addition, the buckling strength and overstressing strength of the fiber-reinforced composite cylindrical skirt have been analyzed using the classical laminate theory and the elastic stability theory of thin shells. The Tsai-Wu failure criterion has been employed to assess the first ply failure, and an overstressing load level factor has been introduced to investigate the failure strength. In addition, a buckling load factor has been introduced to study the buckling strength. Due to critical issue of buckling strength, the effects of the design parameters on the buckling strength have also been investigated in this work.

A practical design example of a fiber-reinforced composite cylindrical skirt has been presented and investigated using the analysis procedure. Some results for this particular design example have been presented and discussed. The following conclusions can be drawn from this study

1. For the present practical optimization design example of a fiber-reinforced composite cylindrical skirt, the best design is $(\theta, h_\theta, h_c) = (42.5°, 1.807\ \text{mm}, 1.877\ \text{mm})$ and the lightweight designs raise in the vicinity of the orientation angle of the angle-ply layers is $\theta = 45°$. This reveals that the fiber-reinforced composite cylindrical skirt is strengthened by symmetrically laminated with both the cross-ply layers [0/90°] and the angle-ply layers [+45/ − 45°].
2. The buckling strength due to axial compressive thrust is the most critical issue in the present practical design example of a fiber-reinforced composite cylindrical skirt. In the future, when similar cylindrical shell structures are designed, buckling strength should be given first priority.
3. The buckling strength of the skirt shell laminated symmetrically with hybrid-layers was found to be greater than that of the skirt shell laminated with pure single layers. In addition, the buckling strength of the skirt shell laminated symmetrically with equal hybrid between the angle-ply layers and the cross-ply layers was found to be greater than that of the skirt shell laminated with

over-weighted hybrid between the angle-ply layers and the cross-ply layers.

In this paper, the HGA optimal design procedure has been employed to determine the best laminate configuration for a fiber-reinforced composite cylindrical skirt with multi-layer sandwich lamination. The results can serve as valuable reference for designers of aerospace vehicles. In the future, this procedure will be applied to find the optimal designs of more complex laminated cylindrical structures, such as multi-layer hybridized sandwich laminated cylinders.

Appendix A

The components of the reduced stiffness matrix $[\bar{Q}]^{(\theta)}$ of the angle-ply layers of a laminate are described as

$$\bar{Q}_{11} = Q_{11}c^4 + 2(Q_{12} + 2Q_{66})c^2s^2 + Q_{22}s^4,$$
$$\bar{Q}_{12} = \bar{Q}_{21} = (Q_{11} + Q_{22} - 4Q_{66})c^2s^2 + Q_{12}(c^4 + s^4),$$
$$\bar{Q}_{22} = Q_{11}s^4 + 2(Q_{12} + 2Q_{66})c^2s^2 + Q_{22}c^4,$$
$$\bar{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12})c^2s^2 + Q_{66}(c^2 - s^2)^2,$$
$$\bar{Q}_{16} = \bar{Q}_{61} = -Q_{22}cs^3 + Q_{11}c^3s - (Q_{12} + 2Q_{66})(c^2 - s^2)cs,$$
$$\bar{Q}_{26} = \bar{Q}_{62} = -Q_{22}c^3s + Q_{11}cs^3 - (Q_{12} + 2Q_{66})(c^2 - s^2)cs.$$

The components of the reduced stiffness matrix $[\bar{Q}]^{(c)}$ of the cross-ply layers of a laminate are described as

$$\bar{Q}_{11} = \bar{Q}_{22} = Q_{11} + Q_{22},$$
$$\bar{Q}_{12} = \bar{Q}_{21} = 2Q_{12},$$
$$\bar{Q}_{66} = 2Q_{66},$$
$$\bar{Q}_{16} = \bar{Q}_{61} = \bar{Q}_{26} = \bar{Q}_{62} = 0,$$

where $c \equiv \cos\theta$, $s \equiv \sin\theta$ and θ is the orientation angle of the layers; Q_{ij} $(i,j = 1,2,6)$ are the components of the lamina stiffness matrix $[Q]$ and are related to the commonly known engineering constants E_L, E_T, G_{LT}, ν_{LT}, and ν_{TL}. These components, Q_{ij} $(i,j = 1,2,6)$, are stated as

$$Q_{11} = \frac{E_L}{1 - \nu_{LT}\nu_{TL}},$$
$$Q_{22} = \frac{E_T}{1 - \nu_{LT}\nu_{TL}},$$
$$Q_{12} = \frac{\nu_{LT}E_T}{1 - \nu_{LT}\nu_{TL}},$$
$$Q_{16} = Q_{26} = 0,$$
$$Q_{66} = G_{LT}.$$

References

[1] Rosato DV, Grove CS. Filament winding: its development, manufacture, applications, and design. New York: Wiley; 1964.

[2] James FF. Structural design and analysis of filament wound rocket motor cases. Technical Report (AD876829); 1970.

[3] Maheshwari M, Grover RL. Development of advanced composite rocket motor case for internal and external load environments. Technical Report (AIAA-Paper-84-1352); 1983.

[4] Hoffman HJ. Kevlar rocket motor cases. CPIA Report CPTR-83-21; 1983.

[5] Peters ST, Humphrey WD, Foral RF. Filament windings-composite structure fabrication. SAMPE Press; 1991.

[6] Francis M. Design, manufacture and test of composite case for ERINT-1 solid rocket motor. AIAA/SAE/ASME/ASEE 19th Joint Propulsion Conference and Exhibit; 1993.

[7] Nshanian YS, Pappas M. Optimal laminated composite shells for buckling and vibration 21;1983:430–7.

[8] Onoda J. Optimal laminate configurations of cylindrical shells for axial buckling. AIAA J 1985;23:1093–8.

[9] Sun G, Hansen JS. Optimal design of laminated-composite circular-cylindrical shells subjected to combined loads. J Appl Mech 1988;55: 136–42.

[10] Sun G. A practical approach to optimal design of laminated cylindrical shells for buckling. Compos Sci Technol 1989;36:243–53.

[11] Hu HT. Influence of shell geometry on buckling optimization of fiber-composite laminate shells. J Pressure Vessel Technol 1991;113: 465–71.

[12] Zimmermann R. Quick optimum buckling design of axially compressed, fiber composite cylindrical shells. AIAA J (Tech, Notes) 1995;33:1993–5.

[13] Xie YJ, Yan HG, Liu ZM. Buckling optimization of hybrid-fiber multilayer-sandwich cylindrical shells under external lateral pressure. Compos Sci Technol 1996;56:1349–53.

[14] Walker M, Reiss T, Adali S. Minimum weight design of composite hybrid shells via symbolic computation. J Frankin Inst 1997;334B: 47–56.

[15] Walker M, Reiss T, Adali S. Multi-objective design of laminated cylindrical shells for maximum torsional and axial buckling loads. Comput Struct 1997;62:237–42.

[16] Smerdov AA. A computational study in optimum formulations of optimization problems on laminated cylindrical shells for buckling I. shells under axial compression. Compos Sci Technol 2000;60: 2057–66.

[17] Smerdov AA. A computational study in optimum formulations of optimization problems on laminated cylindrical shells for buckling II. Shells under external pressure. Compos Sci Technol 2000;60: 2067–76.

[18] Adali S, Verijenko VE, Richter A. Minimum sensitivity design of laminated shells under axial load and external pressure. Compos Struct 2001;54:139–42.

[19] Sadagopan D, Pitchumani R. Application of genetic algorithms to optimal tailoring of composite materials. Compos Sci Technol 1998; 58:571–89.

[20] Goldberg DE. Genetic algorithms in search, optimization, and machine learning. Reading, MA: Addison-Wesley; 1989.

[21] Michalewicz Z. Genetic algorithms + data structures = evolution programs, 3rd ed. Berlin: Springer; 1996.

[22] Vanderplaats GN. Numerical optimization techniques for engineering design with application. ; 1993. International Edition.

[23] Rao SS. Engineering optimization theory and practice. New York: Wiley; 1996.

[24] Rohan PSH, Michael TL, Robert AS, Lloyd AS. Optimization techniques for a computer simulation of a pastoral dairy farm. Comput Electr Agric 1998;19:129–53.

[25] Jones RM. Mechanics of composite materials. New York: McGraw-Hill; 1975.

[26] Agarwal BD, Broutman LJ. Analysis and performance of fiber composites, 2nd ed. New York: Wiley; 1990.

[27] Timoshenko SP, Gere JM. Theory of elastic stability, 2nd ed. New York: McGraw-Hill; 1961.

[28] Vinson JR. The behavior of shells composed of isotropic and composite materials. Dordrecht: Kluwer Academic Publishers; 1993.