Burning analysis of star configuration

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October 27, 2000

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1 Introduction

The design of solid propellant grain that provide neutral burning is important to optimize rocket motor performance. The star configuration have been widely used to achieve this goal. In this report, I will present an analysis of the burning comportement of star shape as well as parameter recommandation to achieve better performance.

2 Geometric definition

The star could be characterize by seven independant variable as defined in figure 2. As every star points are identical, only one is necessary for the analysis.

$$\begin{aligned} w &= \text{Web thickness} \\ r_1 &= \text{Radius} \\ r_2 &= \text{Tip radius} \\ R &= \text{External radius} \\ \eta &= \text{angle} \\ \varepsilon &= \text{Secant fillet angle} \\ N &= \text{Number of star points} \end{aligned}$$

3 Analysis

In this section, an expression for the perimeter of the star will be developp for each burning zone as a function of the web thickness burned (w_x).

3.1 Zone 1

The perimeter in the zone one could be divide in three sections. Starting by the right, we have the section before the radius r_1, which have a radius equal to $R - w + w_x$. The length of this section is then: $(R - w + w_x)(\pi/N - \varepsilon)$.

Then, we have the perimeter of the arc of initial radius r_1. The angle will remain constant to a. The length is then: $(r_1 + w_x)a$.

The third section is more complicated. The lenght of the line starting at the end of the radius r_1 and crossing the vertical line will be evaluated

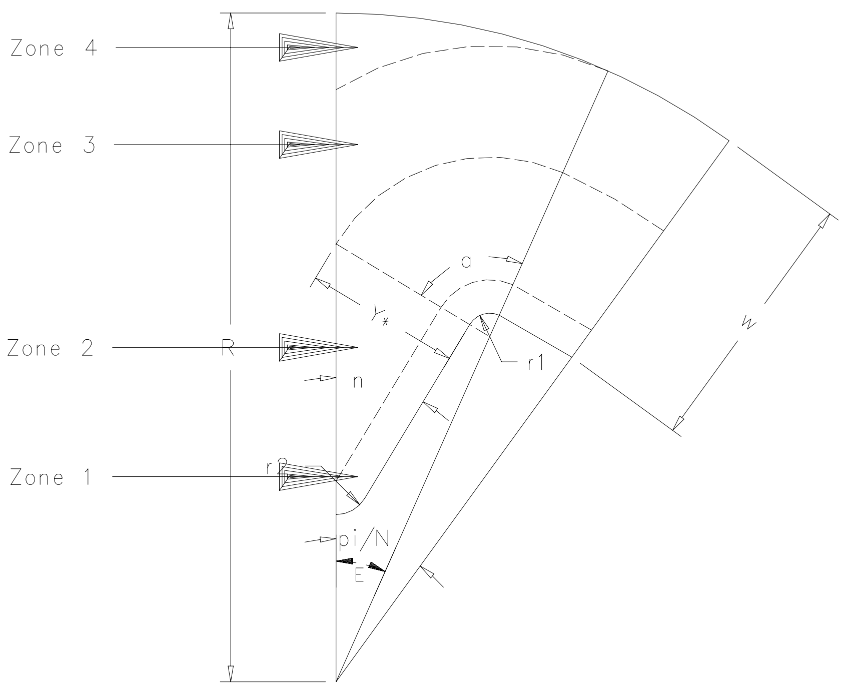

Figure 1: Geometric definition of star.

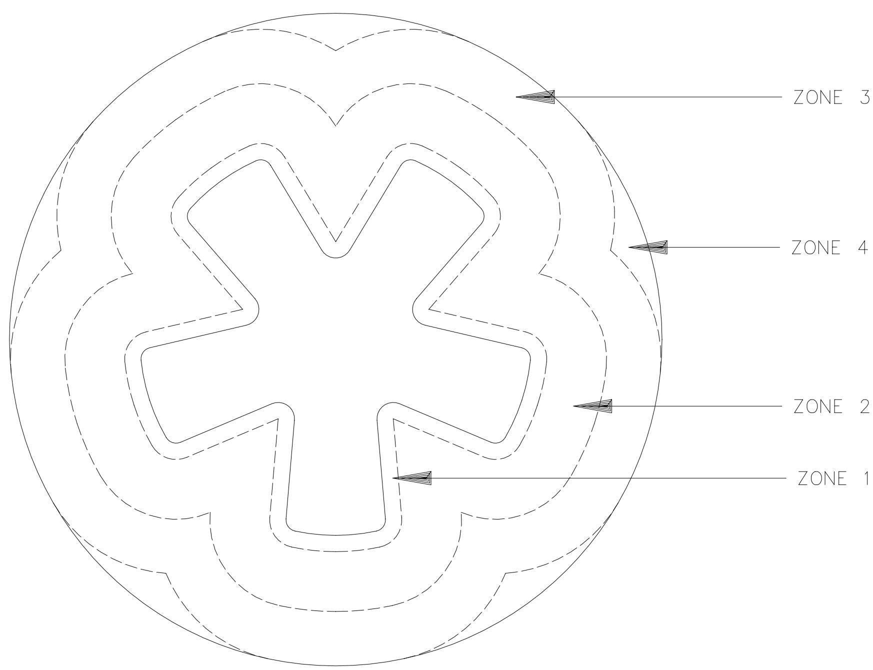

Figure 2: Burning zone of the star configuration.

first. Then, the perimeter of the radius r_2 will be add to the result, and the length of the line starting at the beginning of the radius will be substract.

In order to determine the length, refer to the figure 3. The lenght L we are looking for will be equal $x + y$.

$$\begin{aligned} b + z &= (R - w - r_1)\sin\varepsilon \\ x &= (r_1 + w_x)\tan\eta \\ \cos\eta &= \frac{r_1 + w_x}{z} \\ \sin\eta &= \frac{b}{y} \\ \sin\eta &= \frac{(R - w - r_1)\sin\varepsilon - \frac{r_1 + w_x}{\cos\eta}}{y} \\ L = y + x &= (R - w - r_1)\frac{\sin\varepsilon}{\sin\eta} - \frac{r_1 + w_x}{\cos\eta\sin\eta} + (r_1 + w_x)\tan\eta \end{aligned}$$

We could now simplify this equation using two trigonometric identity:

$$\begin{aligned} \sin^2\eta - 1 &= -\cos^2\eta \\ \tan\eta &= \frac{1}{\tan\left(\frac{\pi}{2} - \eta\right)} \end{aligned}$$

$$\begin{aligned} L &= (R - w - r_1)\frac{\sin\varepsilon}{\sin\eta} + (r_1 + w_x)\left[\frac{\sin^2\eta - 1}{\cos\eta\sin\eta}\right] \\ &= (R - w - r_1)\frac{\sin\varepsilon}{\sin\eta} + (r_1 + w_x)\left[\frac{-\cos\eta}{\sin\eta}\right] \\ &= (R - w - r_1)\frac{\sin\varepsilon}{\sin\eta} + (r_1 + w_x)\left[\frac{-1}{\tan\eta}\right] \\ &= (R - w - r_1)\frac{\sin\varepsilon}{\sin\eta} - (r_1 + w_x)\tan\left(\frac{\pi}{2} - \eta\right) \end{aligned} \tag{1}$$

We could now determine the length of the arc and how much we should substract from the length L. Refer to figure 4 for the variables.

$$\text{Arc length} = (r_2 - wx)\left(\frac{\pi}{2} - \eta\right)$$

$$x = \frac{r_2 - wx}{\tan\eta}$$

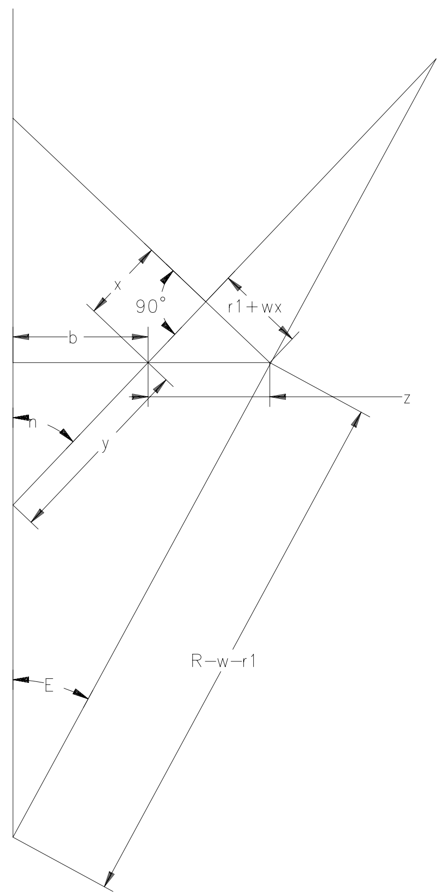

Figure 3: Determination of the length L.

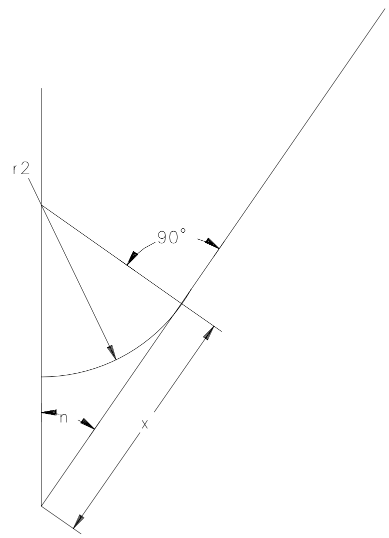

Figure 4: Arc of radius r_2.

We have now the complete expression of the perimeter of the star as a function of web burned (w_x) in the zone one. This expression is valid for $0 < w_x < r_2$.

$$
\begin{aligned}
\frac{S}{2N} &= (R - w + w_x)(\frac{\pi}{N} - \varepsilon) + (r_1 + w_x)a + (R - w - r_1)\frac{\sin\varepsilon}{\sin\eta} \\
&- (r_1 + w_x)\tan(\frac{\pi}{2} - \eta) + (r_2 - w_x)(\frac{\pi}{2} - \eta) \\
&- (r_2 - wx)\tan(\frac{\pi}{2} - \eta) \\
&= (R - w + w_x)(\frac{\pi}{N} - \varepsilon) + (r_1 + w_x)a + (R - w - r_1)\frac{\sin\varepsilon}{\sin\eta} \\
&- (r_1 + r_2)\tan(\frac{\pi}{2} - \eta) + (r_2 - w_x)(\frac{\pi}{2} - \eta)
\end{aligned} \tag{2}
$$

We could now determined the first derivative of this expression to evaluate if it is progressive, regressive or neutral.

$$
\frac{\delta S}{\delta w_x} = 2N\left[\frac{\pi}{N} - \varepsilon + a - \frac{\pi}{2} + \eta\right] \tag{3}
$$

We could verify that:

$$a = \frac{\pi}{2} - \eta + \varepsilon$$

Our expression become:

$$\frac{\delta S}{\delta w_x} = 2\pi \tag{4}$$

The perimeter in zone 1 will always be progressive. So, it is important to minimize the radius r_2 in order to switch as fast as possible to the zone 2.

3.2 Zone 2

The expression for the perimeter in the second zone is almost the same as in the zone one. The difference is that the radius r_2 had vanish and the expression reduce to a simpler one:

$$\begin{aligned}\frac{S}{2N} &= (R - w + w_x)(\frac{\pi}{N} - \varepsilon) + (r_1 + w_x)a + (R - w - r_1)\frac{\sin\varepsilon}{\sin\eta} \\ &- (r_1 + w_x)\tan(\frac{\pi}{2} - \eta)\end{aligned} \tag{5}$$

The derivative of this expression is:

$$\begin{aligned}\frac{\delta S}{\delta w_x} &= 2N\left[\frac{\pi}{N} - \varepsilon + a - \tan(\frac{\pi}{2} - \eta)\right] \\ &= 2N\left[\frac{\pi}{2} - \eta + \frac{\pi}{N} - \tan(\frac{\pi}{2} - \eta)\right]\end{aligned} \tag{6}$$

As we could see in this expression, the progressivity in zone 2 is determined by the angle η and by the number of star point N. It is independant of the angle ε.

The zone 2 will be predominant during the motor burn time and we would like to provide neutrallity in this zone. Neutrality is obtain when the derivative of the perimeter is equal to zero. This lead to the following equation:

$$\frac{\delta S}{\delta w_x} = 0 = \frac{\pi}{2} - \eta + \frac{\pi}{N} - \tan(\frac{\pi}{2} - \eta) \tag{7}$$

Which reduce to the following implicit equation of η as a function of N:

$$\eta = \frac{\pi}{N} - \tan\left(\frac{\pi}{2} - \eta\right) + \frac{\pi}{2} \tag{8}$$

Solution of this equation give values of the angle η to obtain neutrality in zone 2 as a function of the number of star points.

N	η (deg)	π/N (deg)
3	24.55	60.00
4	28.22	45.00
5	31.13	36.00
6	33.53	30.00
7	35.56	25.71
8	37.31	22.50
9	38.84	20.00

It is important to note that when the angle $\eta < \pi/N$, a secant fillet $\varepsilon < \pi/N$ will be necessary to prevent star point from overlapping. In general, ε should always be smaller that π/N.

3.3 Zone 3

The perimeter in the zone 3 begin when $w_x = Y*$. The angle a become progressivly smaller when propellant burned. Perimeter could be expressed like this:

$$\frac{S}{2N} = (R - w + w_x)\left(\frac{\pi}{N} - \varepsilon\right) + (r_1 + w_x)\left[\varepsilon + \arcsin\left(\frac{R - w - r_1}{r_1 + w_x}\sin\varepsilon\right)\right] \tag{9}$$

The derivative of this expression become:

$$\frac{\delta S}{\delta w_x} = 2N\left[\frac{\pi}{N} + \arcsin\left(\frac{R - w - r_1}{r_1 + w_x}\sin\varepsilon\right) - \frac{w_x(R - w - r_1)\sin\varepsilon}{(r_1 + w_x)^2\sqrt{1 - \frac{(R-w-r_1)^2\sin^2\varepsilon}{(r_1+w_x)^2}}}\right] \tag{10}$$

It could be demonstrate that the perimeter is progressive in this section. It would be interesting to eliminate the zone 3 in order to keep neutrality as long as possible.

The condition for the elimination of zone 3 is:

$$Y* = (R - w - r_1)\frac{\sin\varepsilon}{\cos\eta} - r_1 = w \tag{11}$$

This equation reduce to:

$$\sin\varepsilon = \frac{w + r_1}{R - w - r_1}\cos\eta \tag{12}$$

Now, the angle ε is determine by the web thickness w, the radius r_1 and the angle η. As η was determine by the number of star points N and the radius may be dictate by technical decision, the web thickness w will determine ε.

3.4 Zone 4

The analytical solution of the perimeter in the zone 4 could be found with the help of the cosinus law:

$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

The perimeter is then:

$$\begin{aligned}\frac{S}{2N} = (r_1 + w_x)\Bigg[\varepsilon + \arcsin\left(\frac{R - w - r_1}{r_1 + w_x}\sin\varepsilon\right) - \pi \\ + \arccos\left(\frac{(r_1 + w_x)^2 + (R - r_1 - w)^2 - R^2}{2(r_1 + w_x)(R - r_1 - w)}\right)\Bigg]\end{aligned} \tag{13}$$

4 Design example

In this section, a star configuration will be design with the theory developp in the previous sections for a motor of $3inch$ internal diameter.

The goal is to have a perimeter that will remain as constant as possible to mainatin neutrality. It will also be interesting to minimize the number of star points in order to reduce the difficulty to cast the propellant. We could also try to optimize the volumetric loading.

First of all, we could determine the number of star points. In order to maximize the quantity of matter, the angle ε should be equal to π/N. In order to obtain this condition, the angle η should be larger than π/N.

If we refer to the table of the angle η in function of N, to obtain neutrality in zone 2, we must choose $N = 6$ to have $\eta > \pi/N$.

Three conditions are now determine:

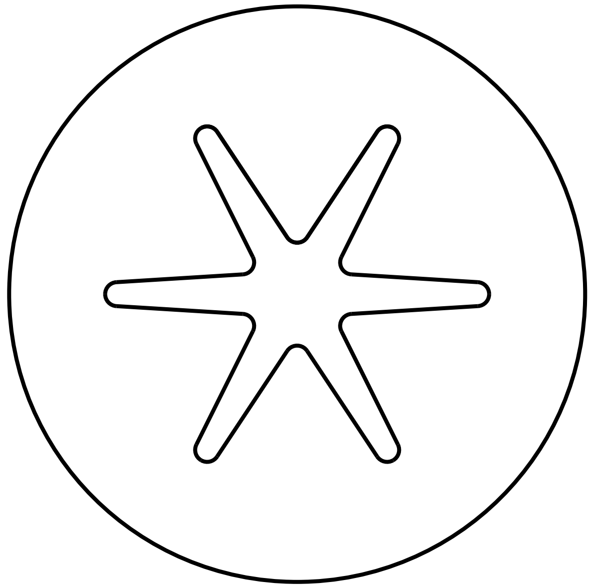

Figure 5: Resulting star configuration for the 3 inch motor.

$$N = 6$$

$$\eta = 33.53deg$$

$$\varepsilon = 30deg$$

We must now found the web thickness w and radius r_1 that fit the conditions. A radius $r_1 = 1/16in$ is reasonable technically.

The equation to be solve is the following:

$$sin\varepsilon = \frac{w + r_1}{R - w - r_1} \cos\eta$$

The value of w that solve this equation is:

$$w = 0.500$$

The seven independant variable are now fixed. The resulting shape could be seen in figure 5.

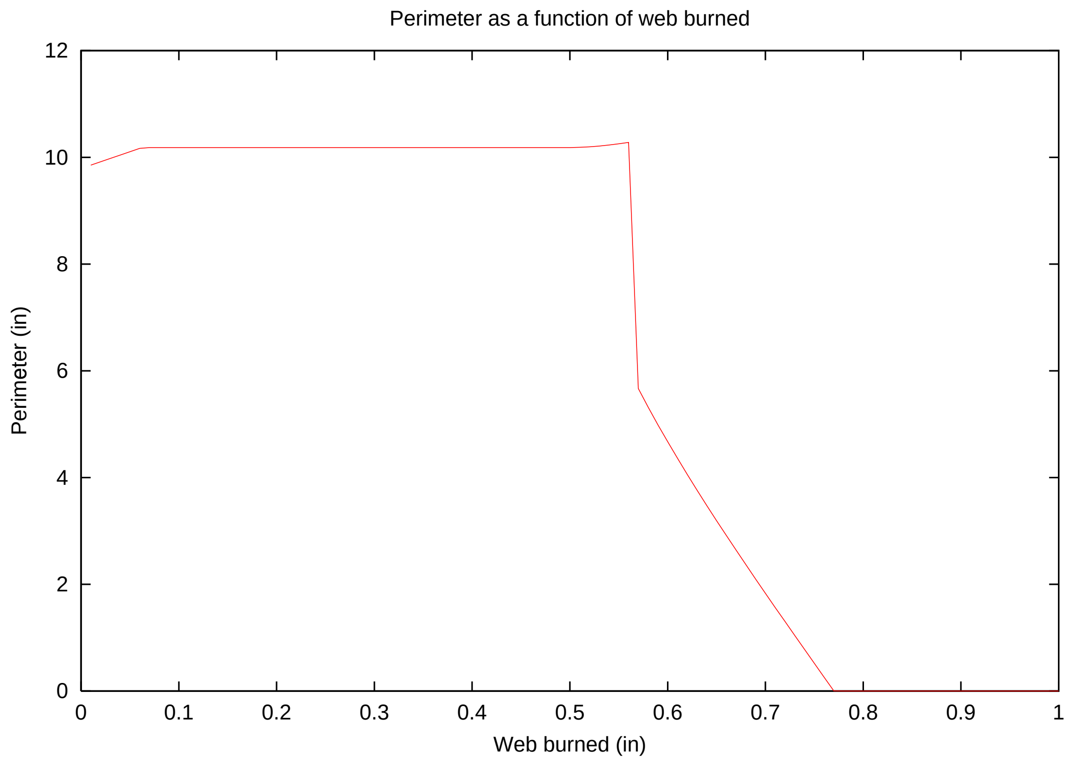

Figure 6: Graphic of the perimeter as a function of web burned.

With the functions developp in the report, the evolution of the perimeter as a function of the web burned could be plot.

5 conclusion

The star configuration offer the possibility to design rocket motor that works at almost constant pressure. It is then possible to optimize on case thickness and throat diameter in order to obtain the best performance.

References

[1] NASA SP-8076, *Solid Propellant Grain Design And Internal Ballistics*, March 1972