Estes Industries Technical Report TR-3

ALTITUDE TRACKING

Single Station Tracking

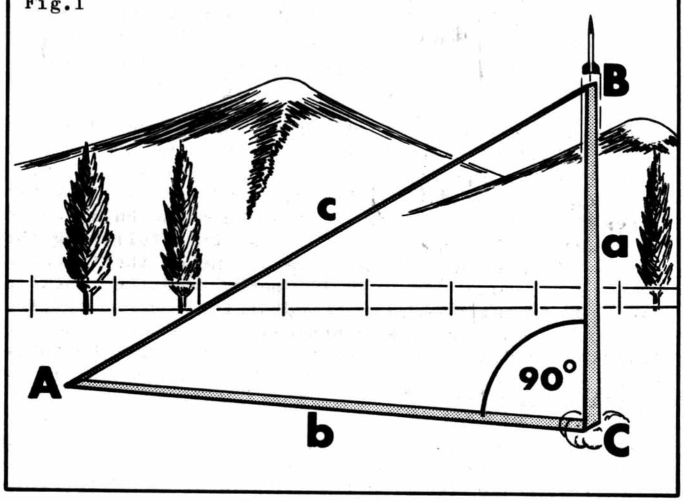
Every Rocketeer asks the question: "How high did it go?" However, previously few model rocketeers had the facilities to determine altitudes with any reasonable degree of accuracy. Some have attempted to find the altitude achieved by their rockets by the use of a stop watch, but this method is so highly inaccurate that the computed altitude may fall anywhere within 200% of the actual altitude. Several years of experience among model rocketeers have proven that optical systems are the only practical means for finding altitudes with any reasonable degree of accuracy.

The use of an optical tracking system requires the use of mathematics. The particular field of mathematics which is used the most in altitude computation is trigonometry. While this field is normally considered an advanced high school subject, any rocketeer can master its basics and apply them to his rocketry activities. If the rocketeer masters a few simple processes, he is ready to solve almost any problem in altitude computation.

One of the first principles of trigonometry is that all of the angles and sides of any triangle can be found if any three of its parts, <u>including</u> one side are known. Now every triangle has six parts: three angles and three sides. So if we know two angles and one side, we can find the other angle and the other two sides.

In determining the height of a rocket we collect two types of data: Distances and angles. This data is used to create a triangle which is a model of the lines which would join the tracker and the rocket, the rocket and a point directly below it on the ground, and the point on the ground and the tracker.

Fig.1



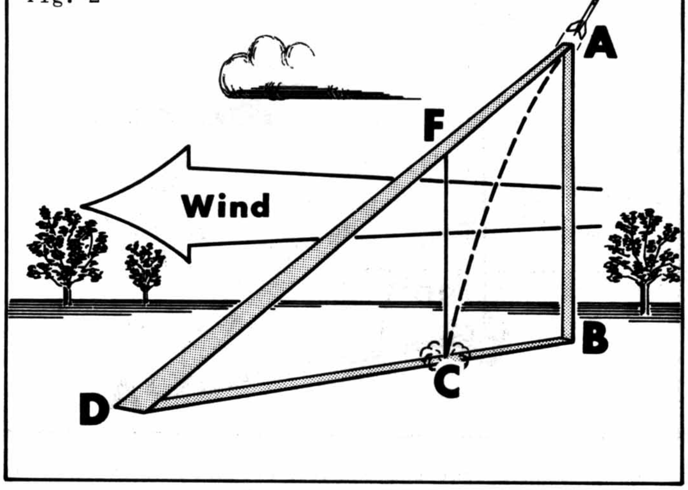
In the diagram above, point A represents the tracking station, B the rocket at its maximum altitude, and C a point on the ground directly below the rocket. The angle formed by the lines at C is then a right angle or 90°. Since there are 180° in the angles of a triangle, if we know angle A, we can find angle B, since $B = 180° - (A + C)$, or $B = 90° - A$. (In trigonometry, a capital letter represents an angle, a small letter represents a side. The small letter "a" will always be used to represent the side opposite angle A, "b" the side opposite B, etc. Two capital letters together represent a distance. Thus BC represents the distance from angle B to angle C, or side "a."

At the firing range, A is found by the tracker when he locks his scope at the rocket's peak altitude. If we now know the distance from A to C, or side b of the triangle, we can find side c and side a. Side a is the one in which we are interested: It is the height of the rocket. This of course assumes that angle C is a right angle.

Now if we only use one tracker, we have the problem of knowing only one angle and one side. This is not enough information to solve the other sides of the triangle. However, we can guess at one of the unknown angles, and obtain a good approximation of the height achieved by the rocket.

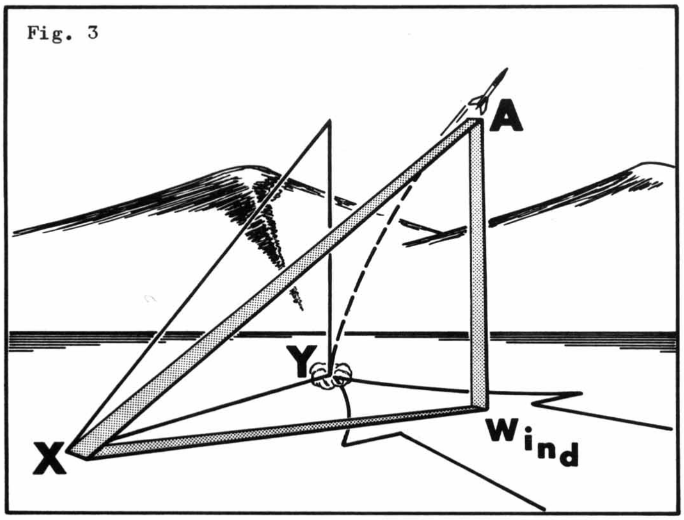
If only one elevation tracker is used, it is a good idea to station it at a right angle to the wind flow. For example, if the wind is blowing to the west, the tracker should be either north or south of the launcher. In this way we will keep the angle at C as close to a right angle as possible. By experimenting with a protractor and a straight edge, the rocketeer can demonstrate why the error would be less if the tracker is on a line at a right angle to the flow of the wind.

Fig. 2



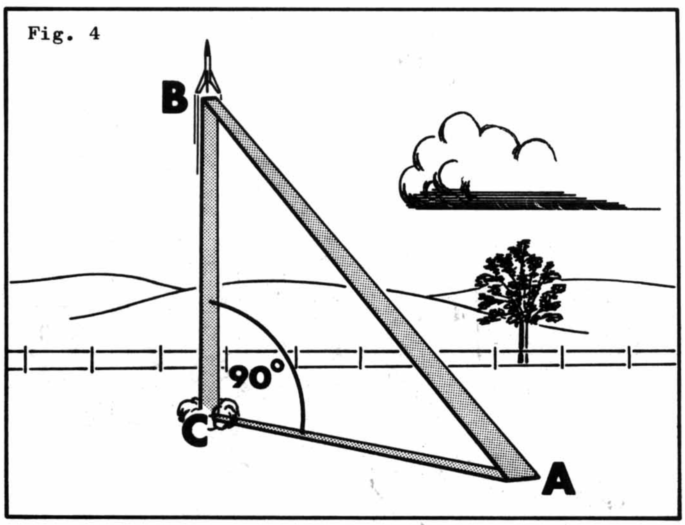
In the diagram above, the wind is blowing from B to D. The rocket is launched at point C, weathercocks into the wind, follows approximately line CA, and at its maximum altitude is at point A. If the tracker is downwind from the launcher, he will see the rocket at point F, and compute the altitude as the distance from C to F. So his computed altitudes will be considerably lower than the true altitudes. On the other hand, if the rocket drifts toward him, his computed altitude will be considerably higher than the true altitude.

However, if the tracker is at point X in figure 3 and the launcher at Y, then the rocket will appear to be at point A as in figure 1, although the distance from the tracker to point A will be slightly greater than the baseline used in computing the altitude, the error will not be nearly as great. Also, the small additional distance will serve to make altitude readings more conservative, as the baseline will be increased.



So by observing the proper relation between wind direction and the position of the tracker, we can generally determine with 90% or better accuracy the altitude the rocket reaches from data given by only one elevation tracker. Of course, the closer the rocket flight is to the vertical, the more accurate will be the figures obtained. Thus on a calm day with a good model, we can approach almost perfect accuracy.

The method used to determine altitude with one tracker is outlined below. Bear in mind that this system assumes that the flight will be almost vertical, if not completely vertical. The rocketeer would do well to master this system before going on to more complex systems, as this method is used as a part of the more involved procedures.



If we assume that the rocket flight is vertical, we can call angle C a right angle (90°). In that case, B is equal to 90° - A (the sum of the angles in a triangle is 180°, half of this or 90° is taken by angle C). Then to find the distance from C to B or the height the rocket reached we take the <u>tangent of angle A</u> (abbreviated tan) <u>times the distance from the tracker to the launcher</u> (side AC of the triangle). For example, if the distance from the tracker to the launcher (baseline) is 250 feet and the angle observed by the tracker at the rocket's maximum height is 62°, we will look in the table of trigonometric functions and find the tangent of 62°. The tangent in this case is 1.88, so we multiply 1.88 times 250 to find our altitude, which is 470'. Altitudes for model rockets are normally rounded off to the nearest ten feet. If the calculated altitude had been 332 feet we would have rounded it off to 330 feet.

Why do we use the tangent to determine altitude? The tangent of an angle is the ratio of the opposite side to the adjacent side, or in other words, the opposite side divided by the adjacent side. In this case, the adjacent side is the distance from the tracker to the launcher, and the opposite side is the distance from the launcher to the rocket's maximum altitude.

Kind souls of many years ago were nice enough to determine the tangents for all angles of right triangles, so we have a table which lists them. Since the tangent of the angle equals the opposite side divided by the adjacent side, or in the case of our first example, 470 divided by 250, by multiplying the quotient times the divisor we find the dividend. In our case, the quotient or tangent is 1.88, the divisor 250, and the dividend 470.

Summary

(1) In single station elevation tracking, we make sure that the line from the tracking station to the launcher is 90° from the direction of wind flow.

(2) The angle of flight is assumed to be vertical.

(3) The tracking scope is locked at the rocket's maximum altitude, the angle read, and the tangent of the angle found.

(4) The tangent is multiplied times the distance from the tracker to the launcher, giving the rocket's altitude.

Two Station Tracking

A higher degree of accuracy is possible when two elevation tracking stations are employed. In such a case, we will have triangles with 2 angles and one side given, enabling us to determine the other parts of the triangle without guesswork.

When using two trackers without azimuth readings the tracking stations are set up on opposite sides of the launcher. Preferably, to obtain the greatest accuracy, the stations should be <u>in line with the wind</u>, unlike the system used in single station tracking. Thus, if the wind is blowing to the south, one station will be north and the other south of the launch area.

The distance between the two trackers is not critical. One might be 100 feet from the launcher and the other 500 feet away. However, for the greatest ease in data reduction, the distances should be equal. For the greatest accuracy, they should be as far apart as possible. A general rule is that the distance from the stations to the launcher should be equal to or greater than the maximum altitude the rocket is expected to achieve.

Some provision should be made to insure that the trackers lock their instruments at the same time. This is one of the greatest problems with any system using more than one station: The one tracker may lock his scope when the rocket appears to him to have ceased rising while the other tracker is still following the rocket. If a phone system is used, one of the trackers or a third party should call "mark," and the trackers should lock their scopes immediately. In the system described here this is especially important, as the elevation readings from the two trackers must be taken at the same point or the altitude computed will be somewhat incorrect.

In this more accurate system we will work with sines instead of tangents. To determine altitude, then, we will first have to find the unknown sides of the triangle, as we have no right angles to work with.

For example, stations A and B are located on a 1000' baseline with the launcher between them. Station A calls in an elevation of 34°, and station B calls in an elevation of 22°. The total of these two angles is 56°, so angle C, located at the peak of the rocket's flight, is equal to 180° - 56°, or 124 degrees. We now have 3 angles and one side to work with. Our first step will be to list the angles and their sines. Since the sine

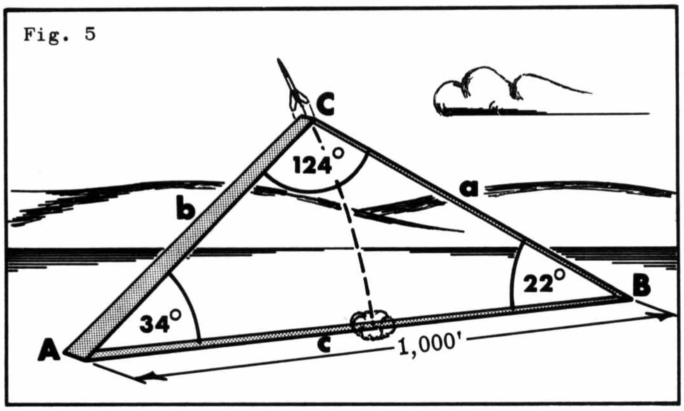


Fig. 5

of any angle greater than 90° is equal to the sine of the supplement of the angle, the sine of 124° is equal to the sine of 180° - 124°, or 56°.

Angle A = 34° Sine A = .5592
Angle B = 22° Sine B = .3746
Angle C = 124° Sine C = .8290

The law of sines states that $\frac{c}{\text{SinC}} = \frac{b}{\text{SinB}} = \frac{a}{\text{SinA}}$

c = 1000', sinC = .8290 Therefore, $\frac{1000}{.8290} = \frac{b}{.3746} = \frac{a}{.5592}$ Pulling out the slide rule, we determine that $\frac{1000}{.8290} = 1205$. So we have a dividend, divisor, and quotient. In solving for side b, our dividend is b, our divisor .3746, and our quotient 1205. To find the dividend we multiply the divisor times the quotient. Now .3746 times 1205 = b, and pulling out the slide rule again, we find that b = 451. The same process is repeated to find side a: $1205 = \frac{a}{.5592}$, a = 1205 x .5592, a = 674'. So we now have the three sides of the triangle.

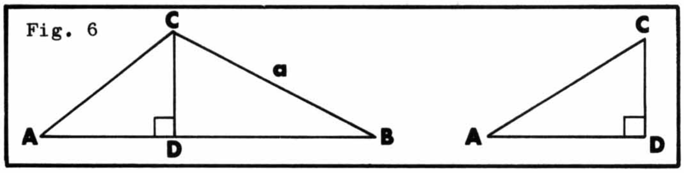


Fig. 6

The altitude of the rocket is then the distance from C to D in the diagram above. The angle formed by the meeting of lines AB and CD is a right angle. Since the sine of an angle in a right triangle is the relation of the opposite side to the hypotenuse, and since we wish to determine the value of the opposite side, we find that the sine of A (34°) is .5592. Hence $.5592 = \frac{a}{451}$, since $\text{SinA} = \frac{\text{opposite side}}{\text{hypotenuse}}$. .5592 x 451 = 252, hence CD = 252', and we now know the altitude reached by the rocket was 252'.

Fortunately, our computations to determine the altitude of the rocket can be simplified. To find the altitude we need only determine one of the unknown sides of the original triangle. So if we find the distance BC (side a) on the triangle, we can multiply it times the sine of B to find the height CD.

So $\frac{c}{\text{SinC}} = \frac{a}{\text{SinA}}$. Since we have found $\frac{c}{\text{SinC}}$ equal to 1205 when C = 124°, $\frac{a}{\text{SinA}} = 1205$. Then 1205 x SinA= side a = 674'. Now we have the one needed side of the triangle, so we can solve for distance CD in the right triangle BCD. The sine on an angle is equal to its opposite side divided by the hypotenuse, so we take the sine of B, which is .3746, times the hypotenuse, or 674' to find the opposite side CD. Thus .3746 x 674 = 252'.

The complete series of computations then would be $\frac{c}{\text{SinC}}$ x SinA = a, and a x SinB = CD. However, if we can combine the formulas to make one formula, we can speed up our work considerably. Now $\frac{c}{\text{SinC}}$ x SinA = A, so we can substitute the expression ($\frac{c}{\text{SinC}}$ x SinA) for a in the formula a x SinB = CD. Our formula then becomes $\frac{c}{\text{SinC}}$ x SinA x SinB = CD. One of the basic rules of algebra tells us that if the dividend is multiplied by a number and the result divided by the divisor, the result is the same as if the division were carried out first and the quotient multiplied by the number. For example, $\frac{10 \times 4}{5} = 8$, and $\frac{10}{5} \times 4 = 8$.

So we can change the expression $\frac{c}{\text{SinC}}$ x SinA x SinB = CD to read $\frac{\text{c x SinA x SinB}}{\text{SinC}}$ = CD. So by performing two multiplications and one division, we can find the altitude of the rocket. The division of SinC into the expression (c x SinA x SinB) can occur at any point, as $\frac{\text{c x SinA}}{\text{SinC}}$ x SinB = CD, and c x $\frac{\text{SinA x SinB}}{\text{SinC}}$ = CD also. This last form of the equation will give the same result as the first, and actually involves the same steps, but is generally easier to use.

Summary

(1) In two station tracking without the use of azimuth readings we station the trackers on a base line approximately equal to twice the altitude the rocket is expected to reach.

(2) The trackers are located <u>in line with the wind</u>.

(3) The scopes are locked at the rocket's maximum altitude, the angles read, and the sines of the angles found.

(4) The altitude is computed by the formula height = $\frac{\text{c x SinA x SinB}}{\text{SinC}}$, when A and B are the angles read by the trackers, c is the baseline distance, and C is the third angle formed by the meeting of the lines of sight of the two trackers.

Sines and Tangents

∠	sin	tan	∠	sin	tan	∠	sin	tan
1	.02	.02	28	.47	.53	54	.81	1.38
2	.03	.03	29	.48	.55	55	.82	1.43
3	.05	.05	30	.50	.58	56	.83	1.48
4	.07	.07	31	.52	.60	57	.84	1.54
5	.09	.09	32	.53	.62	58	.85	1.60
6	.10	.11	33	.54	.65	59	.86	1.66
7	.12	.12	34	.56	.67	60	.87	1.73
8	.14	.14	35	.57	.70	61	.87	1.80
9	.16	.16	36	.59	.73	62	.88	1.88
10	.17	.18	37	.60	.75	63	,89	1.96
11	.19	.19	38	.62	.78	64	.90	2.05
12	.21	.21	39	.63	.81	65	.91	2.14
13	.22	.23	40	.64	.84	66	.91	2.25
14	.24	.25	41	.66	.87	67	.92	2.36
15	.26	.27	42	.67	.90	68	.93	2.48
16	.28	.29	43	.68	.93	69	.93	2.61
17	.29	.31	44	.69	.97	70	.94	2.75
18	.31	.32	45	.71	1.00	71	.95	2.90
19	.33	.34	46	.72	1.04	72	.95	3.08
20	.34	.36	47	.73	1.07	73	.96	3.27
21	.36	.38	48	.74	1.11	74	.96	3.49
22	.37	.40	49	.75	1.15	75	.97	3.73
23	.39	.42	50	.77	1.19	76	.97	4.01
24	.41	.45	51	.78	1.23	77	.97	4.33
25	.42	.47	52	.79	1.28	78	.98	4.70
26	.44	.49	53	.80	1.33	79	.98	5.14
27	.45	.51				80	.98	5.67

For angles of 81° through 89° the sine is .99, the sine of 90° is 1.00. Tangents over 80° are not given, as no sensible data reduction is possible for angles that great.